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CONTROL OF MULTI-SPAN BEAM VIBRATION BY A RANDOM WAVE REFLECTOR

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In this paper, a new wave reflector called random wave reflector (RWR) is introduced for the control of transverse vibration and wave propagation in an infinite, multi-span, simple-support beam. In order to illustrate the theory, RWR is first tested in a simple configuration of controlling plane wave propagation through layers of gas media. Results demonstrate that RWR has great advantages over other types of noise abatement methods. RWR is then applied to control the vibration of the multi-span beam in which the support locations are given in a random manner. Two types of external excitations, an incident vibration wave and an external point force, are considered separately. Transmission loss, localization factor, mode shape and input power flow are used to investigate the effectiveness of RWR. The results show that no vibrational power flow can be tapped into or propagate through a random system at any frequency. The passbands, which always exist in traditional systems, are eliminated in a random system for which much better performance is obtained over a broad frequency range.

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1. INTRODUCTION

Multi-span, simple-support beams are widely used in engineering, and many practical structures can be modelled as such beams. In order to make sure that the structure has a minimal probability of catastrophic failure or malfunction in service, the associated vibration levels and shock response must be predicted and controlled. The problem of vibration control and wave propagation is very important and has been investigated for many decades.

When a simple-support, multi-span beam has uniform physical properties and the spans are of equal length, it is regarded as being uniform or periodic. Each span is an identical subsystem and these spans are coupled together through the rotation at each support. The vibroacoustic response of periodic structures has received much attention in the past and many characteristic properties have been revealed [1–9]. It has been pointed out that the periodic structures can be regarded as structural filters with certain characteristics. Passbands and stopbands of frequency always exist. In a stopband, the propagation wavenumber is complex, energy is attenuated along the path; while in the passband, the propagation wavenumber is purely real and the energy is transmitted freely.

All structures in reality are imperfect due to various reasons. A small imperfection, or disorder, can cause the predictions to be qualitatively incorrect (see below). Disorders may also alter the passband and stopband. The periodic systems with a certain degree of disorder have attracted much attention. When only one of its elements differs from the rest,

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a system is said to contain a single disorder or defect. Bansal [10] studied the effect of such a single disorder in the support location of an infinite and otherwise periodic beam. The disorder was found to cause resonance in the frequency band where vibration is normally attenuated for the periodic structures. Investigations on multiple disorders in an otherwise periodic system have been concentrated on the so-called "periodic disorders". Lee [11] carried out an analysis of the normal mode of linear lattices with periodic impurities, but his method is not suitable for studying the response of engineering structures in general. Bansal [12, 13] analyzed the free-wave propagation through mono-coupled systems with periodic disorders, and considered the response of a periodic beam system having four disordered spans to a convected harmonic pressure field. Bansal [14] went on to investigate the free-wave propagation through a combination of two different semi-infinite, mono-coupled, periodic systems joined together without or through a finite periodic/disordered system.

Random disorders also occur in reality, such as wave scattering in random media [15–18]. When the ideal periodicity is randomly perturbed, the ability of a periodic structure of transmit disturbances indefinitely within the original passbands is reduced. This is known as the localization effect, which was first predicted by Anderson [19] when he considered the transport of electrons in a disordered, three-dimensional, periodic system. In a disordered, periodic structure, the localization factor was introduced by Hodges and Woodhouse [20] and has often been used to quantify the vibration localization. This factor is defined as the average exponential decay rate of the vibration amplitude and measured from one substructure to the next. Cai and Lin [21] developed a new perturbation scheme on the basis of probability theory to calculate the localization factor. Kissel [22] derived the localization factor as a function of the transmission matrix for multi-channel disordered systems using a multiplicative ergodic theorem of Oseledets.

Pierre *et al.* [23] investigated the localization of the free modes of vibration of disordered, multi-span beams by both theoretical and experimental methods. Bouzit and Pierre [24] studied the effects of small randomness in support spacing on the dynamics of multi-span beams of rigid supports. The results showed that the key parameter governing the sensitivity to disorder was the dynamic inter-span coupling. Bouzit and Pierre [25] experimented on the vibration localization in two types of multi-span beams; good agreement was obtained between experimental results and theoretical findings for both perfect and imperfect systems. Ottarsson and Pierre [26] investigated the vibration and wave localization in a nearly periodic string with attached beads. The effect of random disorder of both the bead spacing and the bead mass was studied and was shown to have fundamental differences. Bouzit and Pierre [27] investigated the linear dynamics of nearly periodic, disordered, multi-span beams resting on flexible supports. They showed that the energy conversion phenomenon rendered the mechanism of localization much more complex than in mono-coupled periodic systems. Pierre [28] also reviewed the recent developments in the area of localization in linear structural dynamics problems with particular emphasis on multi-coupled, nearly periodic structures.

To summarize, it is pointed out that passbands always exist in a periodic or nearly perfect structure. However, the location of the passbands can be altered by disorders in the periodic structure. Sometimes, the effect is incidentally beneficial to the control of energy flow through the structure over particular frequency ranges. The purpose of the present study is to explore whether such beneficial effect can be extended over a broad frequency range so that passbands can be eliminated altogether. The disorders studied here will be deliberately arranged instead of being small imperfections existing in otherwise periodic or uniform systems. A similar attempt was made by Huang [29] in the context of converting mean flow energy to wave energy in a supersonic flow duct.

In what follows, we first consider a simple type of random wave reflector (RWR) for the control of plane-wave sound through layers of acoustic media. RWR consists of different wave-carrying media sandwiched in a random sequence of thickness. Since reflected waves originating from different interfaces are not correlated with each other, the total energy of all the reflected waves is found to grow with the number of sandwich layers. After confirming our physical intuition with such simple configuration, we go on to apply RWR to control the vibration and wave propagation in an infinite, multi-span, simple-support beam. Two types of external excitations, an incident vibration wave and an external point force, are considered. The results show that the randomness can eliminate passbands completely. In other words, no vibration power flow can be fed into or propagate through a random system at any frequency.

2. RANDOM WAVE REFLECTOR FOR PLANE SOUND WAVES

The basic principle of RWR is first tested in a simple configuration of noise propagation through and reflection by layers of different gas media. As shown in Figure 1, there are three regions. The left region (a) and the right region (c) have the same basic acoustic medium with acoustical impedance $Z_0 = \rho_0 c_0$ where ρ_0, c_0 are the density and the speed of sound of the medium respectively. The two regions are separated by the multi-layer region (b) where the acoustic impedance alternates between the basic medium denoted by the subscript "0", and another medium denoted by subscript "1" with impedance $Z_1 = \rho_1 c_1$. If the distribution of the layer thickness in region (b) is uniform, we call it a "uniform wave reflector"; if the distribution follows a pattern or periodic variation, we called it a "periodic wave reflector". Note that the uniform wave reflector is a special type of periodic structure. If the layer thickness is given in a random manner, it is called a "random wave reflector" or RWR.

A plane incident wave from the far left, or region (a), is reflected partially at the first impedance discontinuity, and the rest is transmitted and then reflected by all subsequent discontinuities. The reflected waves are also reflected back by impedance discontinuities as they travel to the left, forming complicated patterns of standing waves within region (b). It is not immediately clear whether the energy fluxes of all reflected waves will add up layer by layer, leading to a vanishing amplitude of the final transmitted wave. To test this physical intuition, the dependence of the transmission loss on the distribution of the layer thickness is now investigated.

2.1. METHOD OF SOLUTION

The plane incident wave is given as $I_0 \,\mathrm{e}^{\mathrm{i}(\omega t - k_0 x)}$, where ω is the angular frequency and the common factor $\mathrm{e}^{\mathrm{i}\omega t}$ will be omitted wherever appropriate for the sake of brevity, $k_0 = \omega/c_0$ is

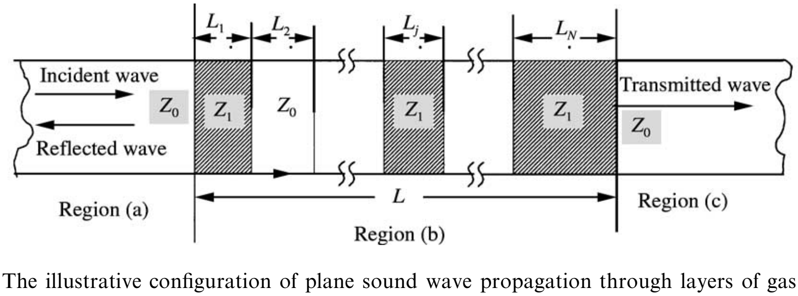

Figure 1. The illustrative configuration of plane sound wave propagation through layers of gas media with alternating impedances.

the wavenumber in the basic medium. For medium "1", the wavenumber is $k_1 = \omega/c_1$. For N layers in region (b), there will be $2N$ waves to be determined. Together with 2 waves in region (a) and the final transmitted wave in region (c), the total number of waves is $2N + 3$. The number of interfaces between layers is $N + 1$. The boundary conditions at each interface are the continuity of particle velocity and sound pressure, giving a set of $2N + 2$ equations. If the amplitude of the incident wave is known, the amplitudes of the other $2N + 2$ waves can be found from the linear set of $2N + 2$ equations.

We number the interface as $j = 1$ at the far left to $j = N + 1$ at the far right, and denote the pressure to the right-travelling waves from interface j to $j + 1$ as $I_j\,\mathrm{e}^{-\mathrm{i}k_j(x-x_j)}$ and the left-travelling waves from $j + 1$ to j as $R_j\,\mathrm{e}^{\mathrm{i}k_j(x-x_j)}$. Note that the phase angles of the complex wave amplitudes are measured relative to the left boundary of a layer, x_j, and $k_j = \omega/c_j$ is the wavenumber for the layer of index j, which is equal to either k_0 or k_1 depending on j. The boundary conditions for each interface, j, are expressed as

$$I_j + R_j = I_{j-1}\,\mathrm{e}^{-\mathrm{i}k_{j-1}l_{j-1}} + R_{j-1}\,\mathrm{e}^{+\mathrm{i}k_{j-1}l_{j-1}},$$

$$(I_j - R_j)/Z_j = (I_{j-1}\,\mathrm{e}^{-\mathrm{i}k_{j-1}l_{j-1}} - R_{j-1}\,\mathrm{e}^{+\mathrm{i}k_{j-1}l_{j-1}})/Z_{j-1},$$

where $j = 2, 3, \ldots, N + 1$, and $l_{j-1} = x_j - x_{j-1}$ is the thickness of layer $j - 1$. Note that the notations for the first interface have to differ from the rest since the phase angle of the incident wave has to be measured relative to the right-hand-side boundary, namely interface $j = 1$. This is equivalent to setting $l_0 = 0$ for $j = 1$:

$$I_1 + R_1 = I_0 + R_0,\ (I_1 - R_1) = \alpha(I_0 - R_0),$$

where

$$\alpha = Z_1/Z_0 = (\rho_1 c_1)/(\rho_0 c_0)$$

is introduced as the impedance ratio. It seems that the linear set of $2N + 2$ equations featuring a matrix of four diagonals will have to be solved simultaneously.

However, it is noted that the situation for the last interface, $j = N + 1$, is different. Denoting the final transmitted wave as $I_{N+1}\,\mathrm{e}^{-\mathrm{i}k_0(x-x_{N+1})}$, we obtain

$$I_N\mathrm{e}^{-\mathrm{i}k_1 l_N} + R_N\mathrm{e}^{\mathrm{i}k_1 l_N} = I_{N+1} + \overbrace{R_{N+1}}^{0}, \qquad I_N\,\mathrm{e}^{-\mathrm{i}k_1 l_N} - R_N\mathrm{e}^{\mathrm{i}k_1 l_N} = \alpha I_{N+1} - \underbrace{\alpha R_{N+1}}_{0}.$$

Since there is no reflection waves from the far right region (c), $R_{N+1} = 0$, these two equations can be solved independently if we specify $I_{N+1} = 1$ instead of the incident wave at the far left, $I_0 = 1$. That way, the equations can be solved in a backward order from interface $j = N + 1$ to 1, and the costly inversion of a large matrix is avoided. Once the wave amplitudes are found, the transmission loss TL defined as $20\log_{10}|I_0/I_{N+1}|$ can be calculated.

2.2. RESULTS AND DISCUSSIONS

We present here a typical example in which the impedance ratio is $\alpha = 0{\cdot}73$, the number of layer is $N = 801$, and the average thickness is $\bar{L} = 0{\cdot}1$ m. Three types of thickness distributions are considered. The first is the uniform type in which all layers have the same thickness of 0·1 m, the second is the periodic type in which the layer thickness repeats the pattern of "0·1, 0·05, 0·1, 0·15", the third type is one in which the thickness is randomly

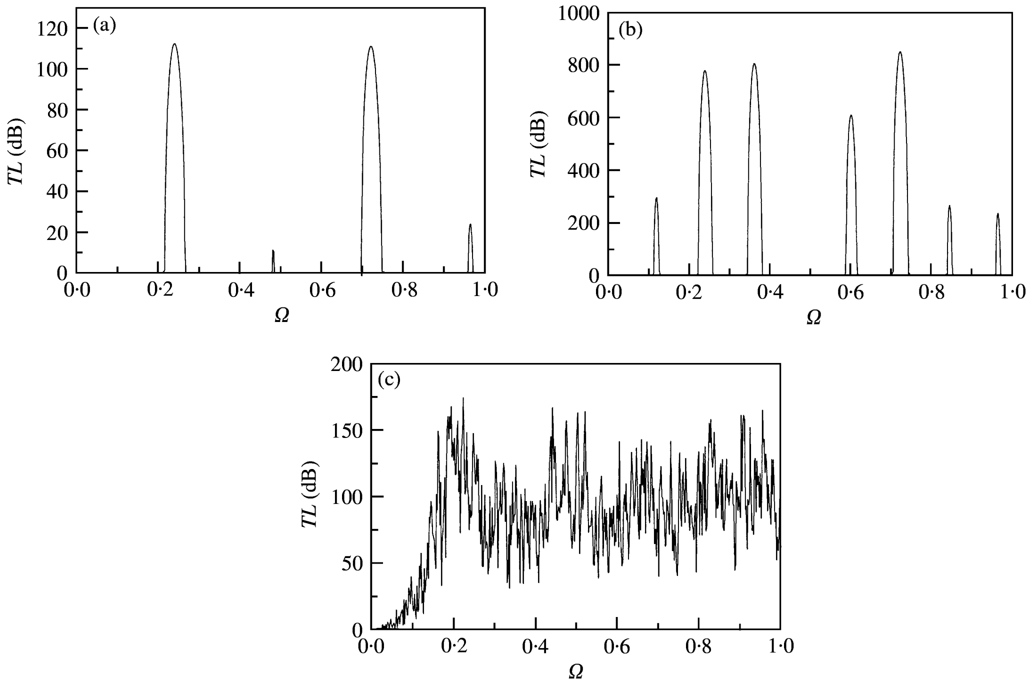

Figure 2. Transmission loss spectra for three types of wave reflections: (a) uniform; (b) periodic, and (c) random.

selected from 0·05 to 0·15 with a uniform probability distribution. The transmission loss (TL) is plotted against the normalized frequency defined as

$$\Omega = \bar{L}f/c_0,$$

where f is the dimensional frequency and c_0 is the speed of sound in the basic medium in regions (a) and (c).

As shown in Figure 2(a), the uniform wave reflector exhibits narrow stopbands around the frequencies of $\Omega = 0{\cdot}24$, 0·72, and to a lesser extent around $\Omega = 0{\cdot}47$, 0·97. Similar performance is shown in Figure 2(b) for the periodic type, where isolated stopbands are found around $\Omega = 0{\cdot}11$, 0·24, 0·37, 0·60, 0·72, 0·85, 0·97. The performance of the random wave reflector (RWR) is shown in Figure 2(c). It can be seen that TL has a random pattern with an average level of about 80 dB. The passbands are eliminated, although the transmission loss under the normalized frequency of 0·2 is not as good. What happens is that the length of the segments is too short to be effective as an individual reflector, and the interfaces can hardly reflect any incident wave. In fact, in the low-frequency band, the random wave reflector can be treated as a composite medium with an equivalent property between the two acoustical media. As a result, the discontinuity is less sudden.

3. THEORY FOR RWR IN MULTI-SPAN BEAMS

We consider an infinite, elastic beam on simple supports as shown in Figure 3. The number of supports is finite, but the uniform beam extends to infinity in both far left and far right regions. Admittedly, all beams in engineering applications are finite in length. The

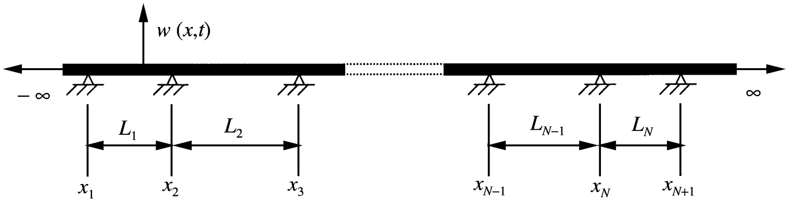

Figure 3. The geometry of infinite, elastic beam with a finite number of simple supports.

adoption of the infinite beam model is justified as follows by considering a problem of vibrational wave incident from the far left. When the random wave reflector works, we except a high transmission loss. The final transmitted wave is very weak. As a result, additional reflection by the right end of a finite beam in reality will have little effect on the result. As far as the far left region is concerned, the wave pattern is already one of standing waves. It makes no difference to the result of TL whether the incident wave is from infinity or partly from the end reflection from the left. As will be shown below, high TL is obtained when the number of the random supports is reasonably high, hence justifying the model of infinite beam.

We consider only the bending motion and neglect shear deformation. If all the spans have identical length, the multi-span beam is regarded as a uniform system; if the locations of these supports are given in a random fashion, the system is called a random wave reflector. The theory is developed below by first considering the general form of solution and boundary conditions.

3.1. GENERAL BEAM VIBRATIONS AND BOUNDARY CONDITIONS

The free flexural vibration is described by

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} = 0,$$

where $w(x, t)$ represents the displacement of the beam, E is the elastic modulus of the materials, I is the cross-sectional second moment of area per unit width, and m is the mass per unit length of the beam. The general solution can be written in the form of travelling waves,

$$w(x, t) = e^{i\omega t} \sum_{s=1}^{4} A_s \, e^{ik_s x},$$

where the four wavenumbers are written in the format of a real constant k_0 determined by the beam property and the driving frequency,

$$k_1 = -k_0, k_2 = ik_0, k_3 = k_0, k_4 = -ik_0, k_0 = (m\omega^2/EI)^{1/4}.$$

k_1 and k_2 are for forward waves, k_3 and k_4 backward waves. k_1 and k_3 represent propagating waves; k_2 and k_4 represent nearfield vibration waves.

The boundary conditions at a simple support of index j are (1) no displacement on the left-hand side denoted by $(x_j)_-$, (2) no displacement on the right-hand side $(x_j)_+$, (3) continuity of slope w_x, and (4) zero bending moment M at the simple support. The four boundary conditions are expressed as

$$w(x_j)_- = w(x_j)_+ = 0, \quad w_x(x_j)_- = w_x(x_j)_+, \quad M(x_j)_- + M(x_j)_+ = 0,$$

which involve four waves in the left span $j-1$, from x_{j-1} to x_j, and four waves in the right span of j defined as $x \in (x_j, x_{j+1})$ with length $L_j = x_{j+1} - x_j$. Denoting the amplitudes of the waves travelling in the jth span by $A_{j,s}$ where the second subscript $s = 1, 2, 3, 4$ specifies one of the four possible waves, and measuring the phase angle relative to the left-hand-side point x_j, one has waves on the jth span expressed as $A_{j,s}\, e^{ik_s(x-x_j)}$. The boundary conditions at the jth interface become

$$\sum_{s=1}^{4} A_{j-1,s}\, e^{ik_s L_{j-1}} = 0,$$

$$\sum_{s=1}^{4} A_{j,s} = 0,$$

$$\sum_{s=1}^{4} k_s A_{j-1,s}\, e^{ik_s L_{j-1}} = \sum_{s=1}^{4} k_s A_{j,s},$$

$$\sum_{s=1}^{4} k_s^2 A_{j-1,s}\, e^{ik_s L_{j-1}} + \sum_{s=1}^{4} k_s^2 A_{j,s} = 0.$$

Note that for the first support, $j = 1$, the phase angle of the far-left waves is measured relative to x_1 and the appearance of the boundary conditions will be slightly different (see below).

For an N-span beam, there are $4(N+2)$ waves to begin with. The extra two accounts for the far left and the far right regions are semi-infinite. There are a total of $N+1$ supports which give $4(N+1)$ equations from four boundary conditions at each support. That means we must know the amplitudes of four waves before we can solve the linear set of $4(N+1)$ equations.Two of the four known waves derived from the fact that, in the far right region, there is no left-travelling waves, $A_{N+1,s=3} = A_{N+1,s=4} = 0$. The other two known waves are found at the far left region, the details of which depend on the excitation method specified.

3.2. EXCITATION BY AN INCIDENT WAVE

If the external excitation is assumed to be a forward propagating vibration wave $I_0\, e^{i\omega t + ik_{s=1}x}$, as shown in Figure 4, the known waves in the far left region is

$$A_{0,s=1} = I_0,\; A_{0,s=2} = 0.$$

It seems that the linear set of $4(N+1)$ equations featuring a matrix with eight diagonals will have to be solved. Again, an easier way of finding the solution is by utilizing the special situations at the far left and the far right regions. At both $j = 1$ and $N+1$, there are four equations with six unknown wave amplitudes. The methodology described in Section 2 is extended here. Instead of trying to find the transmitted wave for a given amplitude of the incident wave, we ask the question of how much incident wave there is in order to produce a transmitted wave of unit amplitude. In the present case of beam vibration, there are now

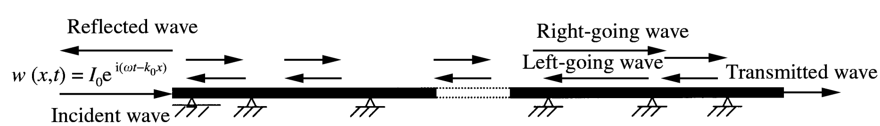

Figure 4. Multi-span beam responding to an incident flexural wave from the far left.

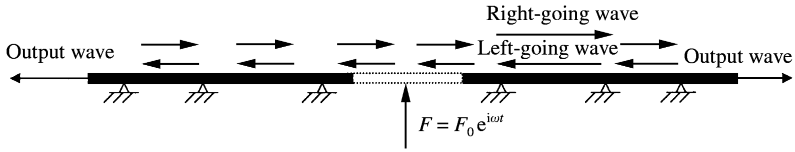

Figure 5. Multi-span beam driven by a point force.

two transmitted waves with amplitudes $A_{N+1,s=1}$, $A_{N+1,s=2}$. The problem is solved in three steps:

Step 1. We solve for the two incident waves which will produce $A_{N+1,s=1} = 1$, $A_{N+1,s=2} = 0$. Once the two transmitted waves are specified, the boundary conditions at x_{N+1} give four equations to be solved for four unknowns, which eventually leads to the solutions for the far left region with four wave amplitudes.

Step 2. We solve for the two incident waves which will produce the alternate transmitted wave pattern of $A_{N+1,s=1} = 0$, $A_{N+1,s=2} = 1$.

Step 3. We search for the coefficients of linear combination of the two solutions which should produce the actual given incident waves of $A_{0,s=1} = I_0$, $A_{0,s=2} = 0$.

This is a problem of solving for a set of two linear equations with two unknowns. The results then give the actual transmitted waves for a given incident wave.

3.3. EXCITATION BY A POINT FORCE

As shown in Figure 5, the external excitation is assumed to be a point force $F = F_0 e^{i\omega t}$ applied at a point of x_F within a certain span. The excitation point is, in fact, a special "support" point which divides the span in question into two. The boundary conditions for the excitation point differ from those of simple supports. Here, we have the continuity of displacement and slope, zero bending moment, and the specified shear force,

$$F = EI \left.\frac{\partial^3 w}{\partial x^3}\right|_{x = x_{F-}} + EI \left.\frac{\partial^3 w}{\partial x^3}\right|_{x = x_{F+}}.$$

The solution procedure for this problem is similar to that of an incident wave described above. The difference lies in step 3. Now the target incident waves in the far-left region are $A_{0,s=1} = 0$, $A_{0,x=2} = 0$. The question of the response of the beam to a unit force excitation is solved by finding the force required to produce the "unit" output waves as a result of the combination of the two previous steps.

4. RESULTS AND DISCUSSION

We first present the results of a basic example, followed by the parametric studies. In the basic example, we use steel as the beam material, so that the elastic modulus $E = 2 \times 10^{11}$ N/m^2 and the density $\rho = 7800$ kg/m^3. The cross-section is circular with a radius of 1 cm. Three types of simple supports are considered similar to the ones used in Section 2. The average distance between adjacent supports is 10 cm for all types, and the total number of supports is either $N = 100$ or 200. The uniform type has N supports distributed by an equal distance. The periodic type has the span length alternating between 8 and 12 cm. The random type has the probability of the separation distance uniformly distributed in the range from 2 to 18 cm. The non-dimensional frequency in this case is defined as $\Omega = \bar{L}\,(m\omega^2/EI)^{1/4}$. The transmission loss (TL), the localization factor, the mode shape, and the input power flow are analyzed. In addition, the effects of the following parameters will be investigated in the parametric studies: the number of supports, the average distance between adjacent supports, and the range of randomness in a random wave reflector.

4.1. TRANSMISSION LOSS AND LOCALIZATION FACTOR

The transmission loss here is defined as

$$TL = 20\log_{10}|A_{0,\,s=1}/A_{N+1,s=1}|.$$

The results of TL are plotted against the normalized frequency Ω in Figure 6. For the uniform and periodic types shown in Figures 6(a) and 6(b), stopbands exist here and there, which contrasts with the broadband performance shown in Figure 6(c) for the random type. The existence of passbands is the inherent property of all periodic and uniform systems which cannot be eliminated easily. The overall reduction of the vibrational energy flow by the random wave reflector is much higher than that by the other types. Similar to the mechanism discussed in Section 2, the better performance of RWR can be attributed to the fact that waves reflected by different supports are not correlated with each other regardless of the driving frequency.

At low frequencies, however, there is a sharp contrast between the performance of RWR used in the multi-span beam and that used in the plane-wave propagation through gas media (see Figure 2). In the latter case, reflection does not happen since the layer thickness is

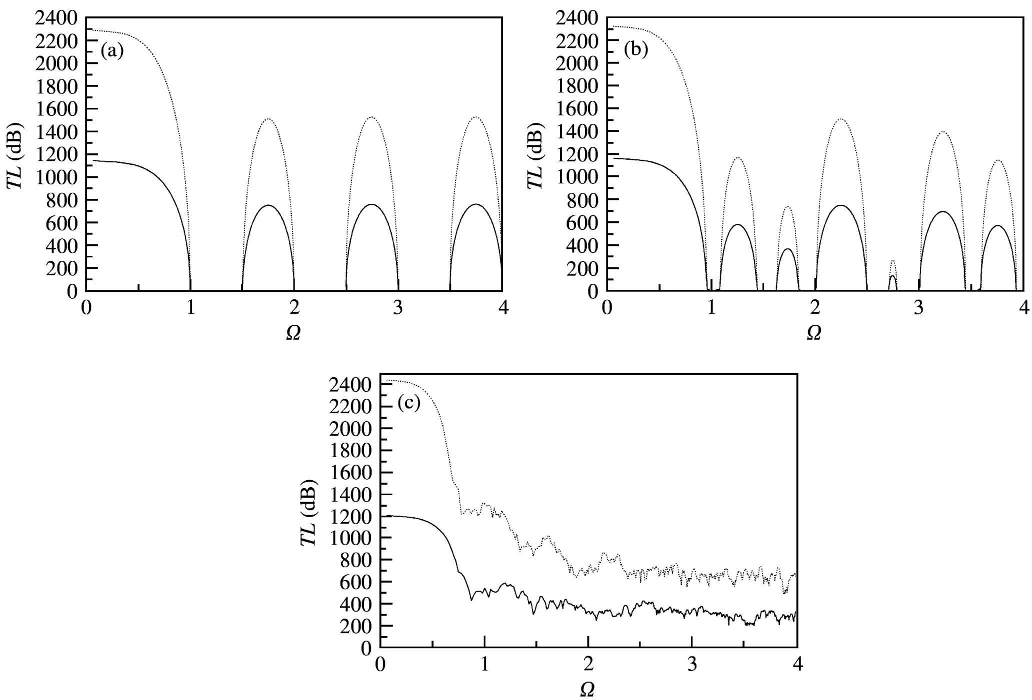

Figure 6. The spectra of transmission loss for (a) uniform; (b) periodic; (c) random wave reflectors with $N = 100$ (——) and $N = 200$ (·········).

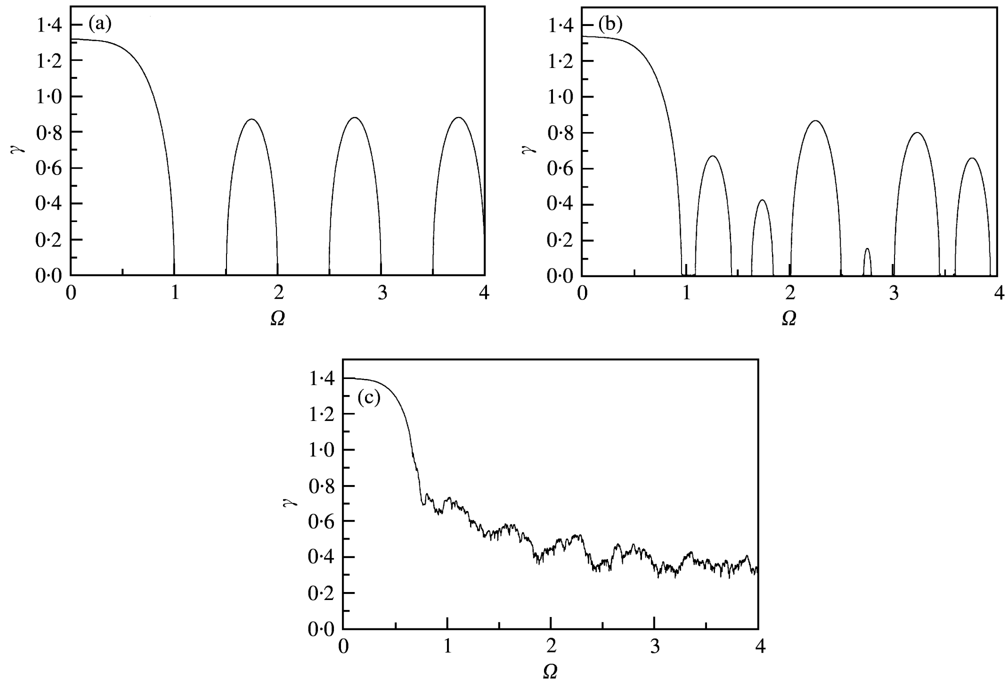

Figure 7. The spectra of localization factor (γ) for the three wave reflections with $N = 100°$: (a) uniform; (b) periodic, and (c) random.

short compared with the wavelength which is very long at low frequencies. Here, for the multi-span beam, the performance of RWR is rather good at low frequencies. The reason is that the beam is a dispersive wave-carrying medium in which the local phase speed $c = \omega k_0^{-1} = \omega^{1/2}\,(EI/m)^{1/4}$ varies with the driving frequency ω. The group velocity of the vibration waves, $c_g = \mathrm{d}\omega/\mathrm{d}k_0 = 2\,(EI/m)^{1/4}\,\omega^{1/2} = 2c$, at which energy flow propagates, is twice as much as the local phase speed. Waves slow down to zero speed towards the DC frequency. In other words, vibrational energy simply cannot propagate at low frequencies, let alone being transmitted through the supports.

The effect of the number of supports is shown in Figure 6. The shape of the TL curves for different N is almost the same. As N increases from 100 to 200, the value of TL doubles. For large N, the amplitude of the transmitted waves decay exponentially, and the rate of logarithmic decay per support is defined as the localization factor,

$$\gamma = \lim_{N \to \infty} \frac{1}{N+1} \ln \left| \frac{A_{0,s=1}}{A_{N+1,\,s=1}} \right|.$$

The results are plotted in Figure 7, which mirrors those of Figure 6.

4.2. MODE SHAPE AND INPUT POWER FLOW

Due to the reflection by the discontinuities, the energy carried by the transmitted waves is reduced along the wave reflector, and the amplitude of the vibration also decreases. Thus, the effectiveness of the wave reflectors can also be illustrated in terms of the mode shape. When the vibration wave $w = I_0\, e^{\mathrm{i}(\omega t - k_0 x)}$ is incident on the wave reflector from the far left region of the beam, the resulting mode shapes are given in Figure 8 for normalized frequencies of $\Omega = 0{\cdot}95, 2{\cdot}1, 4{\cdot}4$. The abscissa now denotes the index of the beam support N, which is somewhat distorted axial co-ordinate of the beam. We observe that the wave amplitudes do not decrease with N for uniform and periodic systems at the passband frequencies, while that in the random system, maintains a robust trend to decrease for all frequencies.

The concept of the input power flow can be used to analyze the forced vibration by an external force. As the beam is driven by the external point force, vibrational power is expected to be fed into the system from the point of action and is transmitted along the beam in both directions. This power input can be compared with the reference case of infinite beam without any support, in which case there is always vibrational power influx at all frequencies. The normalized power input, P', is defined as follows:

$$P = \frac{1}{T}\int_0^T FV\,\mathrm{d}t = \frac{1}{2}\mathrm{Re}\,[F(-\mathrm{i}\omega)\,w_0^*],\;\; P' = P/P_0,$$

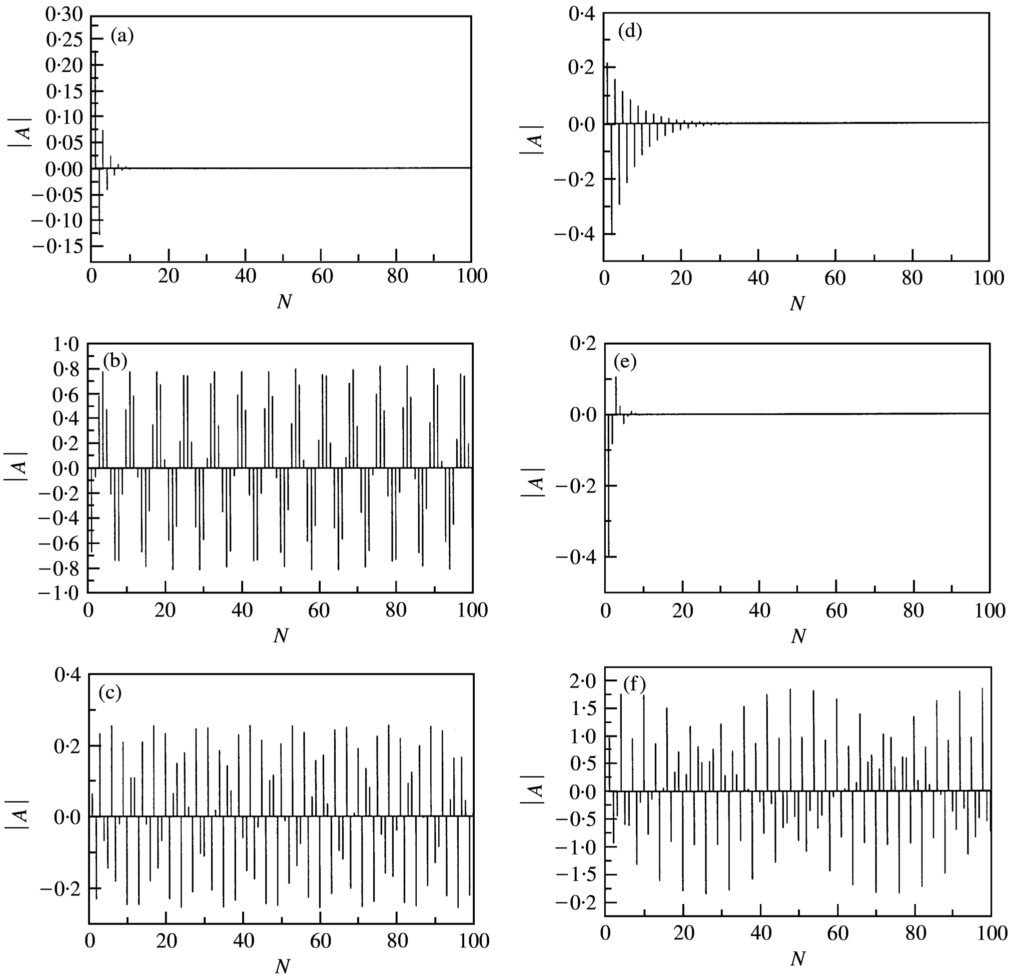

Figure 8. The mode shapes of multi-span, simple-support beams: (a–c) uniform, $\Omega = 0{\cdot}95, 2{\cdot}1, 4{\cdot}4$; (d–f) periodic, $\Omega = 0{\cdot}95$, $2{\cdot}1$, $4{\cdot}4$; (g–i) random, $\Omega = 0{\cdot}95$, $2{\cdot}1$, $4{\cdot}4$.

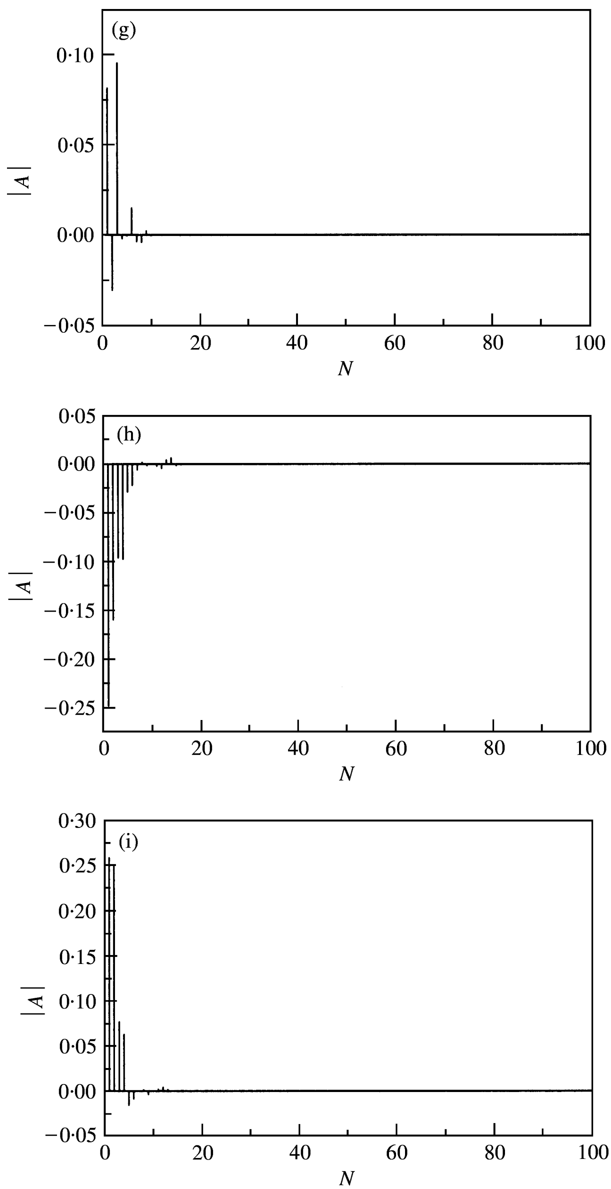

Figure 8. (*Continued*)

where the asterisk denotes the conjugate of complex numbers, and P_0 is the power flow in the reference case. In the stopband, the value of the power flow is expected to be low, in other words, no power can be tapped into the system, such as when an infinite beam is supported randomly. The power flow spectra for the three types of wave reflections with $N = 199$ are shown in Figure 9, which are consistent with the conclusions reached earlier using other criteria of evaluation. It is interesting to note that, at some frequencies, $P' > 1$ for uniform and periodic systems, namely the power input into such beams is higher than that into a infinite beam without any support. This is seen as a form of resonance caused by the supports within the passbands. In contrast, no power flow can be fed into a beam with random supports.

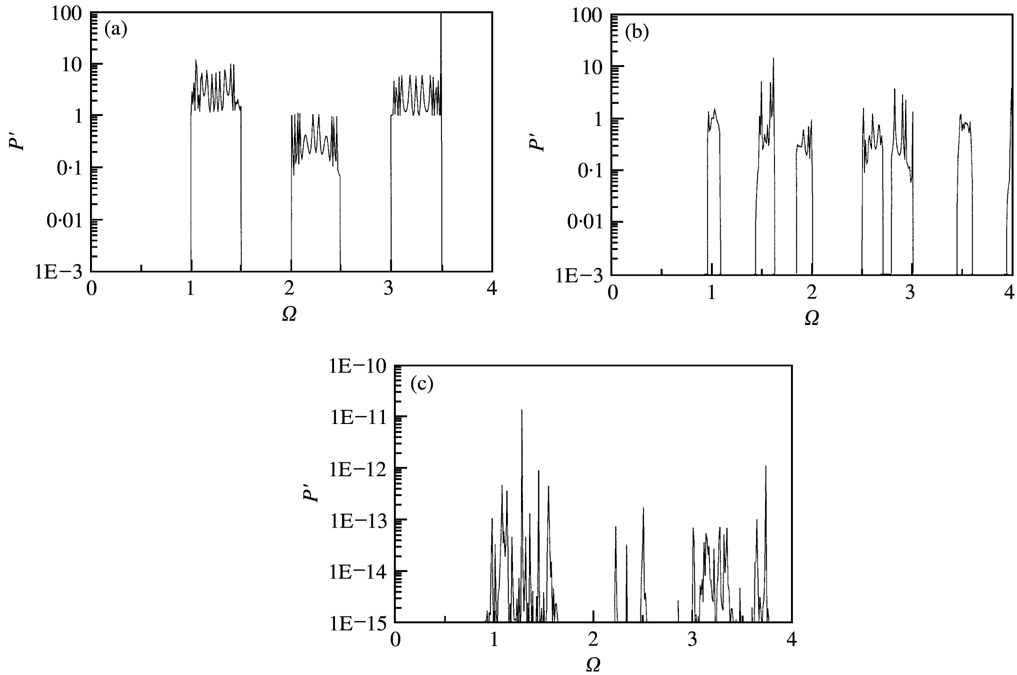

Figure 9. The normalized input power flow for multi-span beams of (a) uniform; (b) periodic; and (c) random distribution of span distances.

4.3. PARAMETRIC STUDIES

The effect of the following design parameters are studied: the range of random distribution of the span distances (Δl), the number of supports (N), and the average separation distance between adjacent supports ($\bar{L}$).

In the example given earlier, the span lengths varies from 2 to 18 cm with a range of $\Delta l = 12$ cm, which is difficult to implement in practice. The results for smaller Δl are given in Figure 10 for the three types of support systems in terms of normalized power flow P'. The total number of supports is $N = 199$ and the mean distance is $\bar{L} = 10$ cm for all systems. The ranges considered are $\Delta l = 1, 2, 3, 4, 5, 6$ cm. We observe that (a) the extent to which the passbands are eliminated in a random system increases with Δl, and (b) the passbands are eliminated more easily at higher frequencies by the increase of Δl. Conclusion (b) is consistent with the results given in reference [25], and can be appreciated by considering the ratio of Δl to the wavelength. At high frequencies, this ratio is high even for small values of Δl, hence better performance.

The effect of the number of supports, N, is given in Figure 11 in terms of the transmission loss for an incident wave from the far left for the span ranging from 5 to 15 cm. N is varied from 25, 50, 100, 150, 200 to 250. The overall value of TL increases with N linearly. In other words, the localization factor is almost constant. Notice, however, that the value of TL oscillates dramatically with frequency when N is very low (25 for instance), which is a reflection of small statistical basis.

The effect of the average span distance, $\bar{L}$, on the transmission loss is given in Figure 12 for RWR with N fixed at 100. The ratio of the lower and higher limits to the average is fixed at 0·5 and 1·5, respectively, while the average distance varies from $\bar{L} = 10$, to 15, 20, 25, 30,

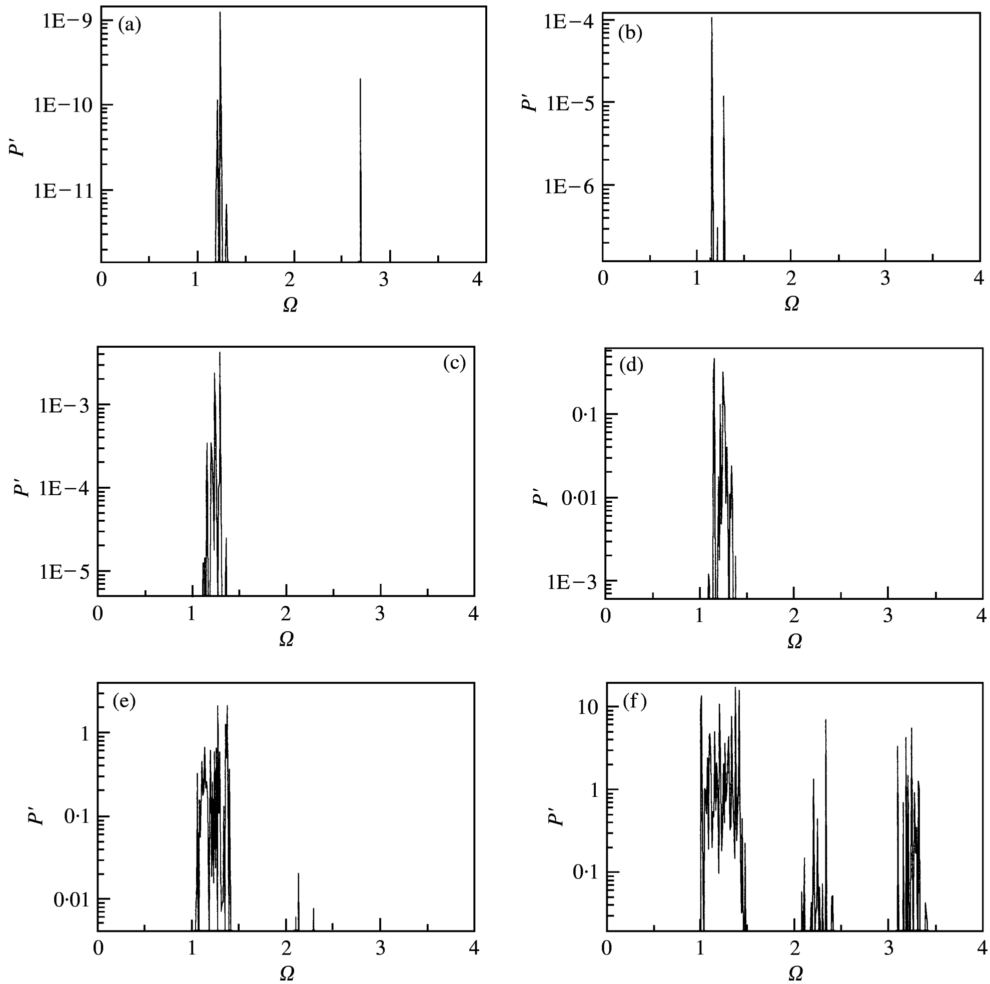

Figure 10. The normalized input power flow for a random beam of various ranges of span distance: (a) $\Delta l = 6$ cm; (b) $\Delta l = 5$ cm; (c) $\Delta l = 4$ cm; (d) $\Delta l = 3$ cm; (e) $\Delta l = 2$ cm; (f) $\Delta l = 1$ cm.

to 35 cm. Several observations are made. First, the level of the sharp stopband near the DC frequency and the level of TL at other frequencies (with TL approximately 300 dB) do not change with $\bar{L}$. Second, the spectral oscillation reduces with $\bar{L}$. Third, the high-peak stopband near the DC frequency narrows as $\bar{L}$ increases. The main conclusion here is that the overall value of TL does not change with the average span distance in a RWR.

5. CONCLUSIONS

A new type of wave reflector, random wave reflector (RWR), is introduced for the control of vibration in an infinite, multi-span, simple-support beam. To help illustrating the theory, RWR is first applied to the control of plane sound waves in gas media. The wave reflector in this case consists of two wave-carrying media sandwiched in a random distribution of thickness. Wave reflection and scattering occur at all interfaces. Due to the de-correlation of

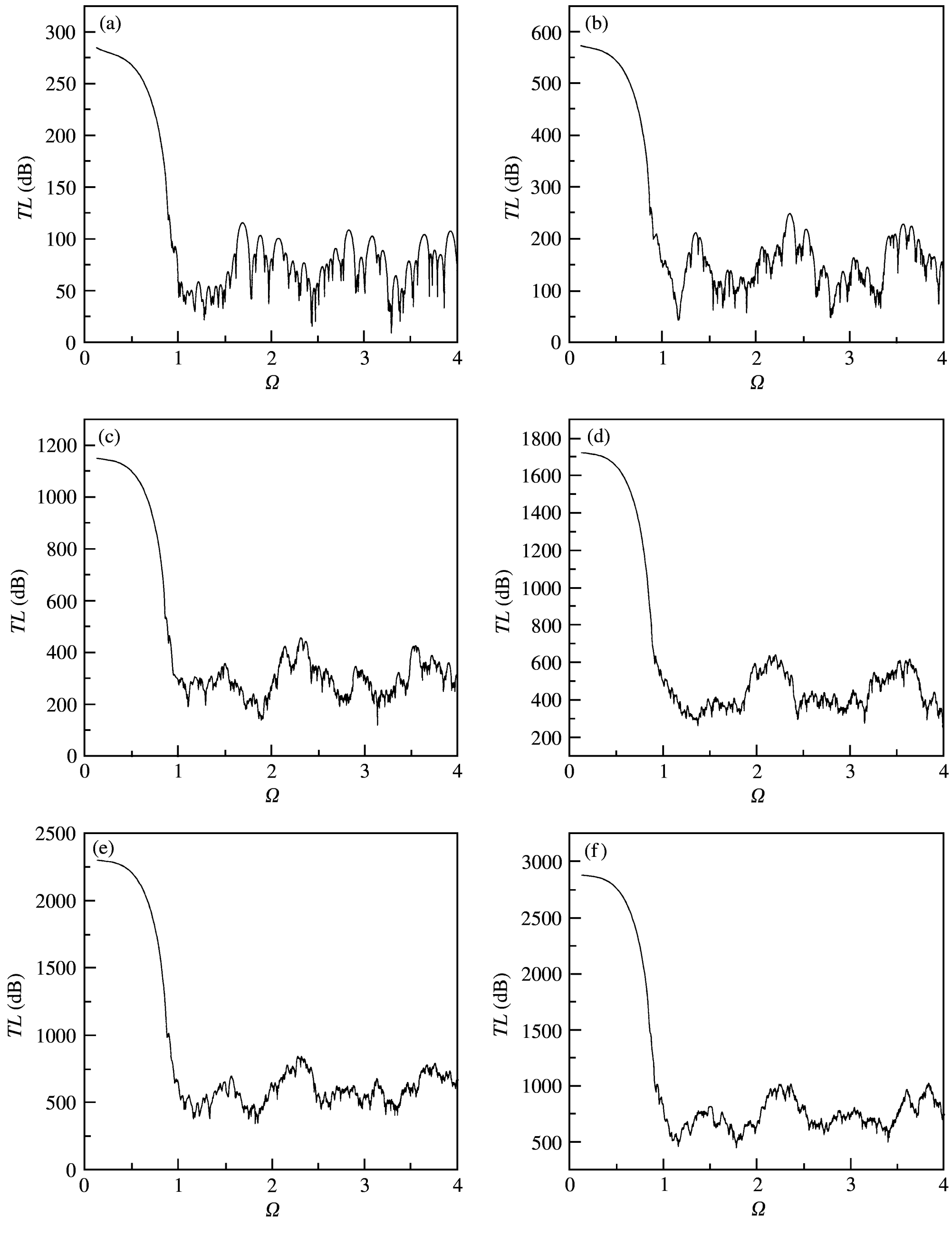

Figure 11. The effect of the number of supports, N, on the transmission loss for an incident wave from the far left: (a) $N = 25$; (b) $N = 50$; (c) $N = 100$; (d) $N = 150$; (e) $N = 200$; (f) $N = 250$.

all reflected waves by the random choice of layer thickness, RWR can eliminate the passbands associated with traditional reactive noise abatement methods. The level of transmission loss is drastically improved, compared with other types of wave reflectors.

The characteristics of RWR are studied in detail for the control of flexural vibrations in an infinite, elastic, multi-span, simple-support beam. The results are consistent with those

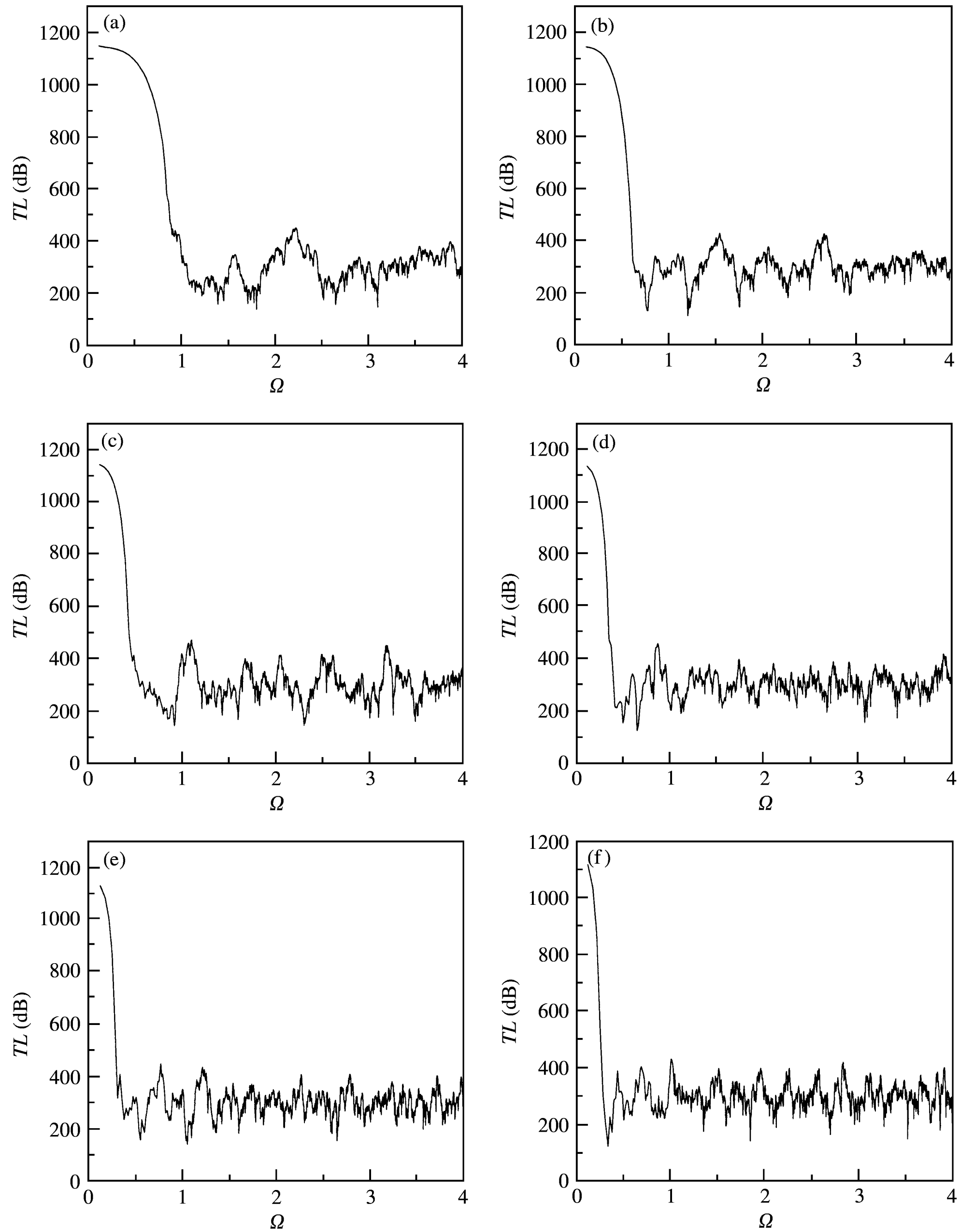

Figure 12. The effect of the average span distance for random wave reflectors with span distance within the range of [$0{\cdot}5\bar{L}$, $1{\cdot}5\,\bar{L}$] where $\bar{L}$ is (a) 10 cm; (b) 15 cm; (c) 20 cm; (d) 25 cm; (e) 30 cm; (f) 35 cm.

found in the studies of the plane sound waves. In addition, the concept of the input power flow from an external force is introduced in the beam study. The results show that no vibrational power can be tapped into a random beam system at all frequencies. The problem of passbands can be completely resolved in a random system, and the performance of RWR is much better than traditional devices. It is anticipated that the concept of

RWR can be carried over to the control of vibration and noise in other more complex structures.

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